Dienstleistungsmanagement
Übung 5

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Besprechung Übungsblatt 4

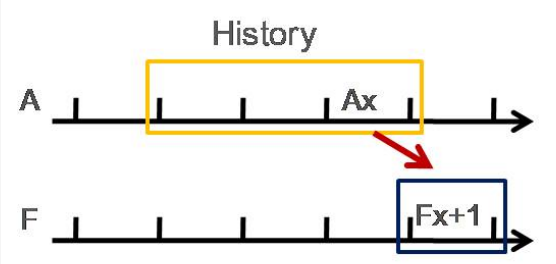
Time Series Models: N-Period Moving Average

Example: Forecast for occupancy rate of a hotel on Saturdays for September 12

Saturday	Date	Period	Occupancy (A)	3-Period Moving Average	Forecast (F)
August	1	1	79		
August	8	2	84		
August	15	3	83	82	
August	22	4	81	83	82
August	29	5	98	87	83
September	5	6	100	93	87
September	12	7			93

Moving Average Period 3: $$MA_3 = \frac{83 + 84 + 79}{3} = 82$$

Moving Average Period 4: $$MA_4 = \frac{81 + 83 + 84}{3} = 83$$



Moving Averages are taken to forecast the occupancy rate for new period.

(Fitzsimmons & Fitzsimmons, 2011)

Lösung Übung 4, Aufgabe 4.B 1)

$$MA_t = \frac{A_t + A_{t-1} + A_{t-2} + \cdots + A_{t-N+1}}{N}$$

Annahme: N = 3

Moving Average Juli: (26 + 27 + 23)/3 = 25,33 (25 Packungen)

Bitte immer den Rechenweg und die Formel aufschreiben!

Time Series Models: Exponential Smoothing: Simple Exponential Smoothing

Exponential smoothing: Used e.g., for demand forecasting, Based on N-period moving average

- Accidental increases or decreases in the average are “smoothed out” (extreme values weight less for forecasting of following periods): Advantage compared to other methods
- Old data is kept in calculation, but influence is reduced steadily
- “Defensive” forecasting method: Forecasting values often lower than actual values

Forecast error of the previous period is put back in next period

A_t = Actual observed value for current period t

S_t = Smoothed value for current period t

S_{t-1} = Smoothed value for previous period

$(A_t - S_{t-1})$ = Forecast error of previous period (Difference of actual and smoothed value of previous period)

α = Smoothing constant; between 0 and 1 (fraction of the forecast error that is added to smoothed value of previous period)

Smoothing value for current period: $S_t = S_{t-1} + \alpha(A_t - S_{t-1})$

(Fitzsimmons & Fitzsimmons, 2011)

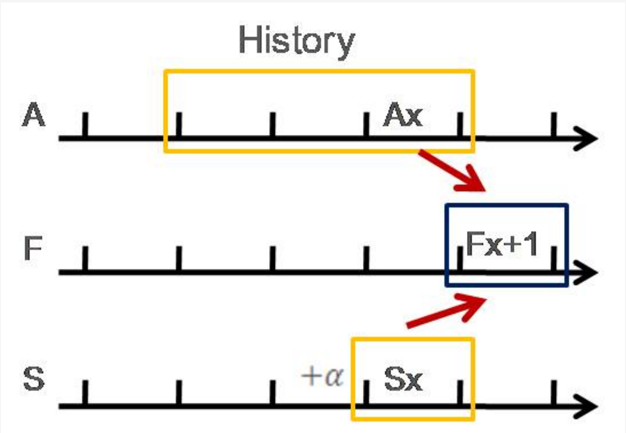
Smoothed value of period t: Used as forecast for following period t+1

F_{t+1} = Forecast for following period t+1

$F_{t+1} = S_t$ (Rounded to an integer)

Smoothed value of period t:

$$S_t = \alpha(A_t) + (1 - \alpha)S_{t-1}$$



(Fitzsimmons & Fitzsimmons, 2011)

Saturday	Date	Period	Actual Occupancy	Smoothed Value	Forecast	Error
		t	A_t	S_t	F_t	$A_t - F_t$
August	1	1	79	79,00		
August	8	2	84	81,50	79	5
August	15	3	83	82,25	82	1
August	22	4	81	81,63	82	-1
August	29	5	98	89,81	82	16
September	5	6	100	94,91	90	10

(Fitzsimmons & Fitzsimmons, 2011)

Lösung Übung 4, Aufgabe 4.B 2)

Alpha = 0,1
Periode 2: $F_2 = S_1 = A_1$
Periode 3: $F_3 = S_2$ (gerundet auf ganze Zahl)

Berechnung geglätteter Wert: $S_t = \alpha(A_t) + (1-\alpha)S_{t-1}$
$S_2 = 0{,}1 * 18 + 0{,}9 * 15 = 15{,}3$
$F_3 = 15{,}3$ runden = 15

Monat	Periode	Tatsächlicher Wert	Geglätteter Wert	Prognose	Fehler
	t	A_t	S_t	F_t	$A_t - F_t$
Januar	1	15	15		
Februar	2	18	15,3	15	3
März	3	22	15,97	15	7
April	4	23	16,67	16	7
Mai	5	27	17,71	17	10
Juni	6	26	18,54	18	8
Juli	7			**19**	

Lösung Übung 4, Aufgabe 4.B 2)

Alpha = 0,5
Periode 2: $F_2 = S_1 = A_1$
Periode 3: $F_3 = S_2$ (gerundet auf ganze Zahl)

Berechnung geglätteter Wert: $S_t = \alpha(A_t) + (1 - \alpha)S_{t-1}$
S_2 = 0,5 * 18 + 0,5 *15 = 16,5
F_3 = 16,5 runden = 17

Monat	Periode	Tatsächlicher Wert	Geglätteter Wert	Prognose	Fehler
	t	A_t	S_t	F_t	$A_t - F_t$
Januar	1	15	15		
Februar	2	18	16,5	15	3
März	3	22	19,25	17	5
April	4	23	21,13	19	4
Mai	5	27	24,06	21	6
Juni	6	26	25,03	24	2
Juli	7			**25**	

Lösung Übung 4, Aufgabe 4.B 2)

Einfluss der Alpha-Werte:

Stellt dar, wie stark Schwankungen des tatsächlichen Wertes beim geglätteten Wert und somit bei der Prognose für die nächste Periode berücksichtigt werden.

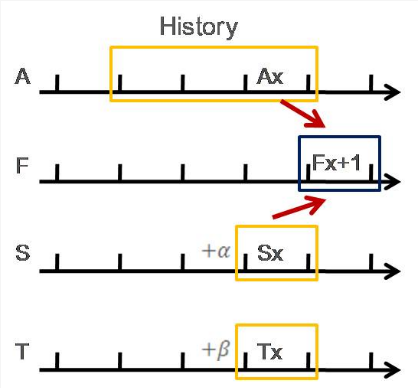
Niedriger Wert für Alpha: Aktuelle Schwankungen werden weniger stark berücksichtigt. Diese haben somit weniger Einfluss auf die Prognose.

Hoher Wert für Alpha: Aktuelle Schwankungen werden stärker berücksichtigt. Diese haben somit einen größeren Einfluss auf die Prognose.

Wahl des Alphas: Abhängig davon, wie stark man aktuelle Schwankungen in die Prognose miteinbeziehen will (Trade-off). Basiert in der Praxis auf Erfahrungen.

Time Series Models: Exponential Smoothing: Trend Adjustment

Trend: Average of change when comparing observed values of previous and actual periods over time



Trend value is added to smoothed value equation:

- T_{t-1} = Smoothed trend value of previous period
- T_t = Smoothed trend value of current period
- β = Smoothing constant (between 0 and 1)

New equation of S_t including smoothed trend value: $S_t = \alpha(A_t) + (1-\alpha)(S_{t-1} + T_{t-1})$

Smoothed trend value of current period: $T_t = \beta(S_t - S_{t-1}) + (1-\beta)T_{t-1}$

Forecast:

$F_{t+1} = S_t + T_t$ (Rounded)

(Fitzsimmons & Fitzsimmons, 2011)

Example

Week	Actual Load Factor	Smoothed Value	Smoothed Trend	Forecast	Forecast Error
t	A_t	S_t	T_t	F_t	$\lvert A_t - F_t \rvert$
1	31	31,00	+ 0,00		
2	40	35,50	+ 1,35	31	9
3	43	39,93	2,27	37	6
4	52	47,10	3,74	42	10
5	49	49,92	3,47	51	2
6	64	58,69	5,06	53	11
7	58	60,88	4,20	64	6
8	68	66,54	4,63	65	3

(Fitzsimmons & Fitzsimmons, 2011)

Lösung Übung 4, Aufgabe 4.B 3)

Alpha = 0,1, Beta = 0,2

Periode 2: $F_2 = S_1 + T_1$

Periode 3: $F_3 = S_2 + T_2$ (gerundet auf ganze Zahl)

Berechnung geglätteter Wert: S_2 = 0,1*18 + 0,9 * (15 + 0) = 15,3

Berechnung Trend: T_2 = 0,2 * (15,3 – 15) + 0,8 * 0 = 0,06

Forecast: F_3 = 15,3 runden = 15

$$S_t = \alpha(A_t) + (1-\alpha)(S_{t-1} + T_{t-1})$$

$$T_t = \beta(S_t - S_{t-1}) + (1-\beta)T_{t-1}$$

Monat	Periode	Tatsächlicher Wert	Geglätteter Wert	Geglätteter Trend	Prognose	Fehler
	t	A_t	S_t	T_t	F_t	$\lvert A_t - F_t \rvert$
Januar	1	15	15	0		
Februar	2	18	15,3 +	0,06 ↘	15	3
März	3	22	16,02 +	0,19 ↘	15	7
April	4	23	16,89	0,33	16	7
Mai	5	27	18,20	0,53	17	10
Juni	6	26	19,46	0,68	19	7
Juli	7	28	20,93	0,84	**20**	8
August	8				**22**	

Aufgabe Overbooking

Overbooking of capacity (e.g., hotel rooms, airplane seats): Selling more seats than total capacity and expecting no-shows

Problems of overbooking:

- If overbooked capacity > number of no-shows:
  - Customers with reservation must be turned away
  - Costs of overbooking: Reimbursement of denied customers

- Overbooking strategy:
  - Minimization of opportunity costs of empty capacity
  - Minimization of costs of passengers with reservation being turned away

(Fitzsimmons & Fitzsimmons, 2011)

Calculation of optimal number of overbookings

- d = Number of no-shows
- P(d) = Probability of a no-show
- x = Number of reservations overbooked
- P(d < x) = Cumulative probability of a no-show

Example: Hotel wants to calculate the loss per night for different numbers of overbooking & no-shows and the optimal number of overbookings

- No-show loss: Loss of contribution for empty room: € 40 p.n.
- Overbooking loss: Penalty costs for guests sent away (need to stay in another hotel): € 100 p. denied guest

No-shows d	Probability P(d)	Reservations overbooked x	Cummulative probability P(d < x)
0	0,07	0	0
1	0,19	1	0,07
2	0,22	2	0,26
3	0,16	3	0,48
4	0,12	4	0,64
5	0,10	5	0,76
6	0,07	6	0,86
7	0,04	7	0,93
8	0,02	8	0,97
9	0,01	9	0,99

(Fitzsimmons & Fitzsimmons, 2011)

Expected number of no-shows = $\sum d * P(d)$

0 * 0,07 + 1 * 0,19 * 2 * 0,22 +…+ 9 * 0,01 = 3,04

Expected opportunity loss (loss of contribution for empty room due to no-show) =
Expected number of no-shows * no-show loss

3,04 * € 40 = € 121,60 p.n.

Avoid some of this loss with overbooking: However: Penalty for guests sent away (if higher number of overbookings than number of no-shows)

- No-show loss: € 40 p.n.
- Overbooking loss: € 100 p. denied guest

Diagonal: Number of no-shows & number of overbookings are the same (no loss: win-win situation)

(Fitzsimmons & Fitzsimmons, 2011)

Loss due to overbookings

		Reservations overbooked (overbookings)									
No-shows	**Probability**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0,07	0	100	200	300	400	500	600	700	800	900
1	0,19	40	0	100	200	300	400	500	600	700	800
2	0,22	80	40	0	100	200	300	400	500	600	700
3	0,16	120	80	40	0	100	200	300	400	500	600
4	0,12	160	120	80	40	0	100	200	300	400	500
5	0,1	200	160	120	80	40	0	100	200	300	400
6	0,07	240	200	160	120	80	40	0	100	200	300
7	0,04	280	240	200	160	120	80	40	0	100	200
8	0,02	320	280	240	200	160	120	80	40	0	100
9	0,01	360	320	280	240	200	160	120	80	40	0
Expected loss	(--)	121,60	91,40	87,80	115,00	164,60	231,00	311,40	401,60	497,40	560,00

(Fitzsimmons & Fitzsimmons, 2011)

Expected loss per number of no-shows and overbookings:

Sum: Expected loss per certain number of no-shows & overbookings * Probability

E.g., total expected loss for 2 reservations overbooked:

$$= 0{,}07 * € 200 + 0{,}19 * € 100 + 0{,}22 * € 0 + 0{,}16 * € 40 + 0{,}12 * € 80 + 0{,}10 * € 120$$
$$+ 0{,}07 * € 160 + 0{,}04 * € 200 + 0{,}02 * € 240 + 0{,}01 * € 280 = € 87{,}80$$

Optimum number of overbookings: Number which minimizes expected loss:

2 overbookings (loss: € 87,80)

Expected gain p.n. from overbooking: Expected loss without overbooking – Expected loss optimum number of overbookings

€ 121,60 – € 87,80 = € 33,80

Critical fractile criterion (to identify best overbooking strategy):

- d = number of no-shows
- x = number of rooms overbooked
- Cu = lost room contribution when customer does not keeps his reservation (underestimation of number of no-shows)
- Co = opportunity loss when no room is available for an overbooked guest (overestimation of number of no-shows)

$$P(d < x) \leq \frac{C_u}{C_u + C_o}$$

$$P(d < x) \leq \frac{€\,40}{€\,40 + €\,100} \leq 0{,}286$$

The next cumulative probability below 0,286 is 0,26 for overbooking of 2 rooms (optimal number of overbookings: 2)

(Fitzsimmons & Fitzsimmons, 2011)

No-shows d	Probability P(d)	Reservations overbooked x	Cummulative probability P(d < x)
0	0,07	0	0
1	0,19	1	0,07
2	0,22	2	0,26
3	0,16	3	0,48
4	0,12	4	0,64
5	0,1	5	0,76
6	0,07	6	0,86
7	0,04	7	0,93
8	0,02	8	0,97
9	0,01	9	0,99

(Fitzsimmons & Fitzsimmons, 2011)

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